Reliability Optimization Model Topological Design for All-Terminal Networks Using a Genetic Algorithm

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Abstract

In the design of communication networks, one of the fundamental considerations is the reliability and availability of communication paths among all terminals. Together, these form the network system reliability. The other important aspect is the layout of paths to minimize cost while meeting a reliability criterion. In this paper, a new heuristic search algorithm based on Genetic Algorithms (GA) is presented to optimize the design of wide area network (WAN) subject to a reliability constraint. The GA is considerably enhanced over conventional implementations to improve effectiveness and efficiency.

Keyword: network design, network reliability, genetic algorithm, all-terminal reliability

1. Introduction

An important part of network design is to find the best way to layout the components (nodes and arcs) to minimize cost while meeting a performance criterion such as transmission delay, the probability of transmission success, throughput, and reliability. This means that the network forms at least a spanning tree. The primary design problem is to choose a set of links for a given set of nodes, to either maximize reliability given a cost constraint, or to minimize costs given a minimum network reliability constraint.

This paper focuses on design of minimum cost reliable networks when a set of nodes and their topology are given, along with a set of possible bi-directional links that connect them. It is assumed that nodes are perfectly reliable and do not fail, and that links have two possible states – good or failed. Links fail independently and repair is not considered. Reliability and cost per unit distance of each link is reliability and cost per unit distance known. The most common objective is to design a network by selecting a subset of possible links so that reliability is maximized and a maximum cost constraint is met.

The network design problem is an NP-hard combinatorial optimization problem [18] where the search space for a fully connected network with \( N \) nodes and \( k \) possible link choices is \( k^{\binom{N}{2}} \). Compounding the exponential growth in number of possible network architectures is that the calculation of network reliability is also an NP-hard problem, which grows exponentially with the number of links. Previous approaches have either been enumerative-based, which are applicable only for small network sizes [15], [24], or heuristic – based which can be applied to larger networks but do not guarantee optimality. Previous heuristic approaches include tabu search [9], [27], genetic algorithms [1], [2], [3], [25], [26], simulated annealing [19], [20] and others [15]. Traditional optimization techniques encompass: 1) mathematical programming, 2) optimality criteria, 3) approximation methods, and 4) steepest descent methods. Optimal structural design has been implemented for many years [21]. In the traditional optimization, the domain is searched using the gradient of the objective function. A limitation arises when the function is not continuous and it is not possible to calculate its gradient. In the 1960s and 1970s Genetic Algorithms (GA) were conceived and developed by John Holland [13]. GA gained popularity after David Goldberg [11], one of Holland’s Students, succeeded in solving a complicated problem presented in his dissertation, GA for optimal design of trusses have been extensively explored.

In recent years, Rajeev and Krishnamoorthy [22] have experimented with GA in the optimal design of trusses, and Ghasemi et al. have demonstrated the suitability of GA to address large trusses with many uncertain variables. Optimization of large trusses using traditional algorithms was presented by Schmit and Lai [28]. However, an advantage of GA is the possibility of considering discrete variable.

The closest version of their research to this paper is network design when maximizing reliability given a cost constraint. They described a heuristic optimization approach using genetic algorithms. The method solves general network design problems to optimality, or near optimality, with respect to reliability. Therefore, this paper uses a genetic algorithm for optimal networks all terminal because of its effectiveness and flexibility in solving many problems combinatorial including those of
reliability design. The paper discusses the GA, network reliability calculation and search strategy. The optimization approach is demonstrated on test problems. It is shown to be flexible, powerful and robust, and is computationally tractable for all-terminal networks.

The design of reliable communication networks is a significant problem in the telecommunications industry. An important stage of network design is to find the best layout of components to minimize cost while meeting a performance criterion, such as transmission delay, throughput or reliability. In this section, we review publication papers related to reliability of network or genetic algorithm.

Deeter and Smith [8] describe a general approach to optimal design of communications networks when considering both economics and reliability. The approach uses a genetic algorithm to identify the best topology of network arcs to collectively meet cost and network reliability considerations. In the result of the study, communications and data networks are becoming increasingly important, ranging from small networks within a building or plant to global network for high-speed data transmission.

Dengiz and Altiparmak [5] have focused on large scale backbone communication network design where the relevant reliability metric is all-terminal network reliability; defined as the probability that every pair of nodes can communicate with each other. In this study, a genetic algorithm (GA) is presented to solve the all-terminal network design problem when considering cost and reliability. The GA is considerably enhanced over conventional implementations to improve effectiveness and efficiency.

Hwang and Cheng [10] focus on network topological optimization with a reliability constraint. The objective is to find the topological layout of links, at a minimal cost, under the constraint that the network reliability is not less than a given level of system reliability. A decomposition method, based on branch & bound is used for solving the problem. In order to speed – up the procedure, an upper bound on system reliability in terms of node degree is applied. A numerical example illustrates, and shows the effectiveness of the method.

Dengiz and Smith [6] have designed of minimum cost reliable communications networks when a set of nodes and their topology are given, along with a set of possible bi – directional arcs to connect them. They are using the genetic algorithm for their work. In the result, it can be seen that an evolutionary approach to optimal network design when considering reliability is effective and flexible. Differences in objective function, constraints and reliability calculation are easy to handle. One difficulty is the number of times that network reliability must be calculated or estimated. An effective approach using GAs. The method solves general network with an efficient and optimal solution. The algorithm can produce both integer and floating – points results. The advantage in considering integer variables lies in being able to compute preliminary sizes for an initial design.

White and Mann [3] presented the optimal network ring design. The problem of network ring design can be described as consisting of three parts: routing, link capacity assignment and ring determination. The researcher presented the GA for solving the problem. GAs have shown themselves to be efficient at searching large problem spaces and have been successfully used in a network design. In the result, this design process has assumed perfectly reliable components, both node and link, which is not the case in reality. The network ring design has been presented that can assist in the design of cost effective networks. The routing, bandwidth allocation and ring design problems can be solved in a single hierarchical step leading to highly cost effective network designs.

Deeter [7] describes a heuristic optimization approach using GAs. The method solves general network design problems to optimality, or near optimality, with respect to reliability. In the study, all terminal network reliability is calculated from the probability that every node in the network is connected to each other. In the result, if a user can be assured of optimal or near optimal results when exerting a small fraction of the computational effort required for enumerative methods there is strong motivation to use a heuristic approach, such as a GA. Besides effectiveness and efficiency, a GA is quite flexible.

Knoak and Smith [4] put forth a new upper bound on the all-terminal network reliability calculation that is more general than bounds in the literature. The bound in this research is applicable for networks where not all links are of the same reliability, a significant relaxation from previous bounds and one that is important to network designers. The new bound is also tighter even when links are of identical reliability, and is calculated in polynomial time. The bound’s performance is demonstrated on over a 100-network problem from the research.

Gargano and He [30] extend the standard analytical and simulation models of reliability networks by considering budget constraints which affect node survivability and thus indirectly total network reliability. A GA and simulation approach was used to develop empirical results for some standard families of networks. The results consist of star graph, wheel graphs, fan graphs, cycle graphs, path graph, complete graph, and complete bipartite graphs. The GA was used to solve this problem with an efficient and optimal solution.

Reeves [29] describes GAs that they have become increasingly popular as a means for solving hard combinatorial optimization problems of the type familiar in operations research. This paper considers what GAs
have achieved in this area, discusses some of the factors that influence their success or failure, and offers a guide for operations researchers who want to get the best out of them. This paper presents the literature review with both enumerative based methods (usually a variation of branch and bound) and heuristic methods (GAs).

This paper presents the use of GA approach to all-terminal design problem. This customization results in an effective and efficient optimization methodology. Furthermore, the approach in this paper will be demonstrated on a large test suite of problem that includes large networks with up to 100 possible links.

Notation

\( Z \) optimized solution of main mathematical problem (minimum network reliability constraint).

\( N \) set of given nodes, with \( |N| \) nodes.

\( L \) set of possible links, with \( |L| \) links.

\((i, j)\) a link between nodes \( i \) and \( j \).

\( p, q \) link\{reliability, unreliability\} for all links; \( p+q = 1 \).

\( G(N, L, p) \) graph \((N, L), \) including \( p \).

\( c_{i,j} \) cost of link \((i, j)\).

\( x_{ij} \) selection status of \((i, j)\); \( x_{ij} = 1 \) if \((i, j)\) is selected, else \( x_{ij} = 0 \).

\( x \) \([x_{1,2}, x_{1,3}, \ldots, x_{1,n}, x_{2,3}, x_{2,4}, \ldots, x_{n-1,n}]\) the set of all \( x_{ij} \).

\( R(x) \) reliability of a network design.

\( R_0 \) minimum network reliability constraint.

\( k_2 \) USD/m (US Dollar/meter) \( k_2 = k_3 = k_1 = 1 \).

\( \delta \) \( 0 \) if \( R(x) \geq R_0 \)

\( 1 \) if \( R(x) < R_0 \)

\( d_{ij} \) transmission delay \((i, j)\).

\( C_\text{max} \) the maximum value of \( c_{ij} \).

\( Z_{\text{max}} \) the largest (worst) value of equation \( Z(x) \) for the current population.

\( \text{Dis}_{ij} \) distance between node \( i \) and \( j \)

2. Statement of the Problem

A communication network can be modeled by a probabilistic graph \( G = (N, L, p) \), in which \( N \) and \( L \) are the set of nodes and links that corresponds to the computer sites and communication connections respectively, and \( p \) is the connection (link) reliability. The networks are assumed to have bi-directional links, parallel (i.e. redundant), transmission delay and therefore are modeled by graphs with non-directed links. Any graph \( G = (N, L) \) is said to be connected if there is at least one path between every pair of nodes. A sub-graph \( G_1 \) of \( G \) is a graph, of which all nodes and arcs are contained in \( G \), i.e., \( G_1 = (N_1, L_1) \) where \( N_1 \subseteq N \) and \( L_1 \subseteq L \). If \( N_1 = N \), the sub-graph \( G_1 \) is called a “spanning sub-graph”. In a connected graph \( G \) of arcs and \( n \) nodes, a tree \( T \) is a spanning tree consisting of \( n-1 \) arcs. The deletion of any edge from a tree results in a disconnected graph. Therefore a connected graph should be at least a spanning tree with \( n-1 \) edges. A communication network topology should be at least a spanning tree and communication network reliability must be greater than the required system reliability value, \( R_0 \).

Objectives and Scope of the work

1. The primary design problem is to choose a set of links for a given set of nodes, to either maximize reliability given a cost constraint, or to minimize cost given a minimum network reliability constraint. Therefore, a heuristic search algorithm based on GAs is developed to find a network topology, which has minimum cost, subject to a system reliability constraint.

2. In a communication network, all-terminal network reliability (also called uniform or overall network reliability) is defined as the probability that every pair of nodes can communicate with each other. This means that the network forms at least a spanning tree.

3. In a communication network, the graph has parallel (i.e. redundant) edges and transmission rate delay.

Assumptions

The following statements define the other problem assumptions.

1. Information of nodes and possible links is given as well as network constraints.

2. Updating the old network consists of 2 methods:
   a. creating the new link with GA;
   b. updating the old network by adding the new nodes and links by GA.

3. Each \( c_{ij} \) and \( p \) are fixed and known.

4. Each link is bi-directional (non-directed links) and parallel in the network.

5. Each link is either operational or failed.

6. The probability of failures of a link is independent of the states of the other links.

7. The transmission has delay.

8. Nodes are perfectly reliable.

Model Formulation

The optimization problem is to minimize

\[
Z = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij} d_{ij} x_{ij}
\]  

Subject to \( R(x) > R_0 \)

The transmission delay is

\[
D_{ij} = x_{ij} k_1 (\text{delay}_{ij})
\]
\[ delay_{ij} = \frac{\text{linkdelay}_{ij}}{\text{linkdelay}_{ij} + \text{nodedelay}} \times 100 \]  
\[ \text{link delay}_{ij} = k_3 \text{Dis}_{ij} d \]  

Where node delay = 10 \( \mu \)s.

The fitness function considering possible infeasible solutions is given by

\[ Z(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} d_{ij} x_{ij} + \delta(c_{\text{sw}}(R(x) - R)) \]  

Therefore, the fitness in this paper is defined as

\[ F(x) = (Z_{\text{max}} - Z(x)) \]

The equation (1) is derived from [1, 2, 3, 5]; equations (2), (3), and (4) are form [31]; and equation (5) is from [4].

When minimizing cost subject to a minimum reliability constraint, the network reliability calculation is necessary to determine feasibility of the candidate design.

3. Solution Algorithm

GA is selected as the heuristic optimization vehicle because of its flexibility and robustness as demonstrated on many NP-hard problems. In GA, the search space is composed of candidate solutions to the problem, each represented by a string, called a chromosome. Each chromosome has an objective function value, called the fitness. A set of chromosomes together with their associated fitness is called the population. This population, at a given iteration of the GA, is referred to as a generation.

There are three main steps in the repeated loop for GA. The first step is the process of selecting strings from the current generation to be parents of the next generation with preference for fitter strings. This is the selection process for reproduction. The second step is the process of combining two selected strings to generate new children strings, which is called crossover. Probabilistically, components of a chromosome are perturbed while generating offspring. This process is called mutation. Together, crossover and mutation comprise reproduction. The third step is the computation of the fitness value using the objective function of each new solution.

Consider an example with five nodes and \( k = 4 \) levels of connection shown in figure 1 and Table 1. Note there are \((5 \times 4)/2 = 10\) possible link for this example but only five are included; the other five are at level of connection \( k = 0 \). Upon examination of the link matrix for figure 1 it is seen that matrix is symmetric. Thus all the information needed to record this particular architecture lies in the upper triangular half of the matrix. This information is placed in a chromosome (vector) of length \((5 \times 4)/2 = 10\) by copying and concatenating each row into the chromosome format shown below:

Chromosome: \{0 1 0 0 2 0 3 1 0 2\}

Note that the only possible values allowed in each position of the chromosome are 0, 1, ..., \( k - 1 \). The solution space of possible network architectures is \( k^{(N(N-1))/2} \).

![Figure 1. Sample Network Architecture.](image)

4. Simulation Study

In order to prove the concept of this research, simulations will be performed by focusing on the reliability optimization model topological design for all-terminal networks using the procedures described in section 3. The proposed algorithm is not yet completed. This leaves for future research.

5. Conclusions

This paper presents a heuristic approach to the design of networks when considering all-terminal reliability formulated as minimizing cost given a reliability constraint. A GA, a heuristic approach, is applied with a constant sized population of candidate network designs maintained throughout the search. We expected that GA strengths are almost non-increasing computational effort, effective optimization, and flexibility. Since GA is an iterative algorithm and improvement is typically diminishing, it may be terminated at any time and still return good results. The
GA is expected to return optimal or near-optimal solutions on every run regardless of problem instance, problem size or random number seed.

**Other Possible Future Projects**

We can use other heuristic searches for solutions to this problem. Swarm intelligence arises as one of the paradigms based on natural systems. The most famous and most frequently used meta-heuristic in this class is the ant colony optimization heuristic. Particle Swarm Optimization (PSO), like the other evolutionary computation algorithms, can be applied to solve most optimization problems and problems that can be converted to optimization problems. PSO have been applied to evolve a neural network and a heuristics search. The method is extremely fast and highly accurate. The relatively small size of the data set indicates the need for further testing and development. Therefore, we can use the PSO in this problem too, and we expect to find out which algorithms are the best algorithms.

**References**